



The Ouroboros Binding

**COMPLETE TO COLLECT****Emerald-Cut Emerald**

Level 4 · 7 Glints



A MANYFOLD FIELD ACCOUNT · AVELWYTHE

The orchard above the landing

By the time the ferry reached the island, the rain had gone east. Silver water ran from the landing steps while a woman rolled apple barrels beneath a carved lintel that now held up the boathouse roof.

The orchard above was not abandoned. Fresh grafts had been tied along trees growing through old walls, and clean sheets moved on a line between two roofless columns. Only the circular ruin at the center belonged to the expedition. The field crew called it **Coilcourt** because their working map needed a name. No inscription did.

Apple roots had shouldered nearly every paving stone out of place. One round stone alone remained level. An old-gold serpent circled its rim, its jaws closed neatly around its own tail. Conservators lowered a recovered fragment into the final gap and eased away their supports.

Vivian turned one shoulder toward the lake and raised both hands, one high and open, the other advancing beneath it. Rain hesitated in the angular space between them and bent sideways. Beyond her hands, Luna knelt at the serpent's head, her notebook braced against one raised knee, and followed the seam without touching it.

The fragment touched. A point of violet fire appeared just beyond Luna's finger. Far below, the lake mist rose into something long-legged and white, took two soundless steps across the water, and came apart. Luna's pencil was moving before the next pulse.



THE WEATHERWORK WAKES A MENAGERIE WITHOUT A CATALOG

At every pulse, the violet point moved to another exact position on the ring. The mist answered. A narrow-winged creature folded into a hound; the hound lengthened into something Luna could not name. She numbered each point and sketched each changing form beside it.

"The rule is fixed," Luna said at last. "The point never leaves the ring, and its present position determines the next one. Surely it must run out of new states."

A raindrop left Vivian's hovering hand and drew a straight wet line through the dust: $y = mx + b$.

"One short formula," Vivian said. "An entire line." Then she marked the interval $0 \leq t \leq 1$. "Bounded. Still infinitely many points."

USE THE WORDS EXACTLY STATE IS NOT RULE

STATE

The complete information needed to determine what happens next.

DETERMINISTIC

The same complete state always produces the same next state.

FINITE-STATE

Only finitely many complete states are possible.

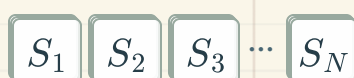
For example, $a_0 = 0$ and $a_{n+1} = a_n + 1$ is deterministic, but its states $0, 1, 2, 3, \dots$ never repeat.

FROM LUNA'S

NOTEBOOK

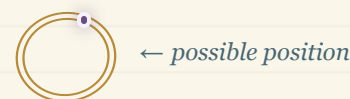
A boundary is not
a count

FINITELY MANY POSSIBLE STATES



Every complete state belongs to
one finite catalog.

A BOUNDED CONTINUUM



One bounded ring
contains infinitely many
possible positions.

FIRST ISOLATE THE VALID THEOREM

I · THE THEOREM LUNA ALMOST HAD

Suppose a deterministic system has
exactly N possible states, producing
 $S_0, S_1, S_2, \dots$

1

Prove that among $S_0, S_1, \dots, S_N$ two states are equal. If $S_i = S_j$ with $i < j$, prove that $S_{i+k} = S_{j+k}$ for every integer $k \geq 0$. Hence, from S_i onward, the sequence repeats every $j - i$ steps.

Use separate paper for all four proofs in this investigation.



THE STATE IN COORDINATES

ONE RULE, REPEATED FOREVER

Luna's sketches preserve the mist. Her coordinate table preserves the binding's exact mathematical state. Write the point after n pulses as $P_n = (x_n, y_n)$.

The point begins at $P_0 = (1, 0)$. Let T mean one pulse. Each pulse applies

$$T(x, y) = \left(\frac{3x - 4y}{5}, \frac{4x + 3y}{5} \right),$$

$$P_{n+1} = T(P_n).$$

Write $d(A, B)$ for ordinary straight-line distance between coordinate points A and B .



The carving repeats its instruction. The point need not repeat its state.

II · MOTION WITHOUT ESCAPE

Use separate paper for your proof.

Call $P_0, P_1, P_2, \dots$ the **orbit**. It **converges** to a point L if $d(P_n, L) \rightarrow 0$ as $n \rightarrow \infty$.

2. Prove each statement.

- $x_n^2 + y_n^2 = 1$ for every n .
- For any points A, B , $d(T(A), T(B)) = d(A, B)$.
- $d(P_n, P_{n+1}) = \frac{2}{\sqrt{5}}$ for every n .

Then explain why the orbit is bounded but does not converge to any point L .

Staying inside one ring is not the same as having only finitely many places to stand.



BINDING REFERENCE

**THE SAME
EXACT RULE**

$$P_0 = (1, 0), \quad P_{n+1} = T(P_n),$$

$$T(x, y) = \left(\frac{3x - 4y}{5}, \frac{4x + 3y}{5} \right).$$

From Part II: every P_n lies on the unit circle, and $d(T(A), T(B)) = d(A, B)$ for all points A, B .

III · EXACT RETURN AND NEAR RETURN

Use separate paper for both proofs.

3.

EXACT RETURN

Prove that no state ever repeats: $P_m \neq P_n$ whenever $m \neq n$. Checking any finite list of states is not enough.

4.

NEAR RETURN

Let $\varepsilon > 0$ be any tolerance, however small. Prove that infinitely many **distinct $n \geq 1$ satisfy $0 < d(P_n, P_0) < \varepsilon$.**

In words: the orbit returns within every chosen distance of its start, infinitely often, without landing exactly there. Explain why this does not contradict Part 1, and identify the missing assumption in Luna's conjecture.

**THE STANDARD OF
EVIDENCE**

A table or picture may suggest the behavior. Your proof must cover the complete infinite orbit.



Out on the lake, the mist unmade one creature and began another. Luna opened to a clean page.

