



The Zero Route



COMPLETE TO COLLECT

The Null Tanzanite

Unique Jewel · 6 Glints

JEWELHOUSE OBSERVATORY LOG

FIND EVERYTHING THE MAP CANNOT SEE

In the **Jewelhouse Observatory**, Vivian compares two proposed signal maps for a dormant Translation Frame. Each map takes a displacement vector $v = (x, y)$ in R^2 and returns a two-number signal. Both maps are linear transformations.

PRECISE LANGUAGE

The kernel

$$\ker(T) = \{v \in R^2 : T(v) = 0\}$$

For a **linear transformation** $T : R^2 \rightarrow R^2$, the **kernel** is the set of all input vectors that T sends to the zero vector $0 = (0, 0)$. Informally, it records every input direction and displacement the map erases.

Finding the kernel means solving $T(v) = 0$, a **homogeneous system**: every output equation equals zero. The zero vector is always in the kernel.

Expedition nickname: a "zero route" is a nonzero vector in the kernel. The standard term is **nonzero kernel vector**.

MAP A

$$T_A : R^2 \rightarrow R^2$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y \\ x - y \end{pmatrix}$$

MAP B

$$T_B : R^2 \rightarrow R^2$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y \\ 2x + 2y \end{pmatrix}$$

YOUR ROUTE

From a system to a theorem

1

solve each
kernel

2

draw what
each map
erases

3

build a
collision

4

prove
collision and
fiber rules

5

prove Vivian's
conjecture

Turn the page for the kernel record. Both map rules are repeated there for reference.



What does each map erase?

Show the system, the complete kernel, and its geometry.

REFERENCE | **Map A** $T_A(x, y) = (x + y, x - y)$

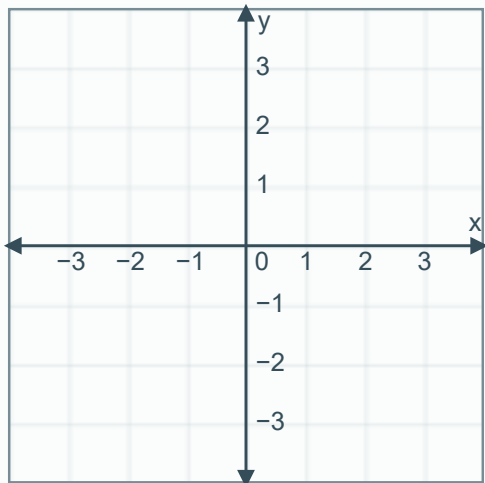
Map B $T_B(x, y) = (x + y, 2x + 2y)$

- For each map, solve $T(v) = 0$. Record the complete kernel in set notation, using a real parameter t if needed. Name its geometry: the origin or a line through it.

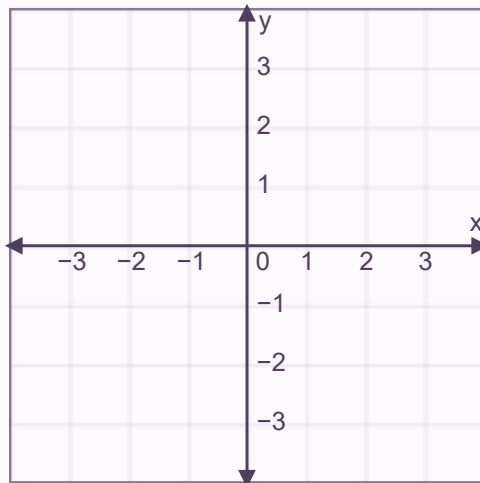
Map	Homogeneous equations and work	$\ker(T)$ in set notation	Geometry
A			
B			

- Draw each kernel. If it is only the origin, mark the origin with a solid dot. If it is a line, draw the entire line across the grid. When the kernel contains a nonzero vector, mark one example and verify by substitution that the map sends it to zero.

Map A



Map B



Substitution check

Show the verification for your marked vector here.

PART 3 • BUILD A COLLISION

Two inputs, one signal

For Map B, find two different inputs p and q that

$$p = (\underline{\quad}, \underline{\quad})$$

$$q = (\underline{\quad}, \underline{\quad})$$



Move every collision to zero

"A list of successful tests is not a proof."

From the examples to the general rule. Let V and W be real vector spaces and let $T : V \rightarrow W$ be linear. Then $\ker(T) = \{v \in V : T(v) = 0_W\}$, where 0_W is the zero vector in W .

One-to-one

For a map $T : V \rightarrow W$, T is **one-to-one** (injective) when different inputs always give different outputs. Equivalently,

$$T(p) = T(q) \Rightarrow p = q.$$

Fiber

For $T : V \rightarrow W$ and $s \in W$, the **fiber over** s is the set of all inputs that produce s :

$$T^{-1}(\{s\}) = \{v \in V : T(v) = s\}.$$

Here $T^{-1}(\{s\})$ denotes the **preimage** of $\{s\}$; T need not have an inverse function. If $a \in V$ and $S \subseteq V$, then $a + S = \{a + k : k \in S\}$ is the **translate of** S by a .

- Let $T : V \rightarrow W$ be linear, and let $p, q \in V$. Prove both directions of $T(p) = T(q)$ if and only if $p - q$ is in $\ker(T)$. Now suppose the fiber over $s \in W$ is nonempty. Choose $a \in V$ with $T(a) = s$, and prove $T^{-1}(\{s\}) = a + \ker(T)$.

$$T(p) = T(q) \Leftrightarrow p - q \in \ker(T).$$

You may use: $T(p - q) = T(p) - T(q)$.

$\Rightarrow$ **Start with** $T(p) = T(q)$

$\Leftarrow$ **Start with** $p - q \in \ker(T)$

Fiber proof.

Show both directions: if $T(v) = s$, then $v - a \in \ker(T)$; if $v = a + k$ with $k \in \ker(T)$, then $T(v) = s$.



Why zero is enough

Use the collision rule below as your theorem-building tool.

CONJECTURE TO PROVE

A linear transformation T is one-to-one if and only if $\ker(T) = \{0\}$.

Tool from Part 4: $T(p) = T(q) \Leftrightarrow p - q \in \ker(T)$

- Using the collision rule, prove both directions of Vivian's conjecture. Then identify which proposed map is one-to-one and state precisely what the kernel records about the input information a linear transformation erases.

Continue on separate paper if needed.

One-to-one $\Rightarrow \ker(T) = \{0\}$

$\ker(T) = \{0\} \Rightarrow$ one-to-one

Which proposed map is one-to-one? _____

In one precise sentence, what does the kernel record?

Vivian's standard: cover every input, not only the vectors tested in the two maps.

