



Action Between Frames



COMPLETE TO COLLECT
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Level 2 · 4 Glints

THE CAMERA SHOULD NOT CHANGE THE MOTION

A puck travels from $x = 0$ at $t = 0$ to $x = 1$ at $t = 1$. A camera divides the same one-second history into N equal time intervals.

If the camera records more frames, should the action of the motion itself change?

ONE MOTION, SEVERAL MEASUREMENTS

The history stays fixed

The puck still travels from 0 to 1 in one second.

The frame count changes

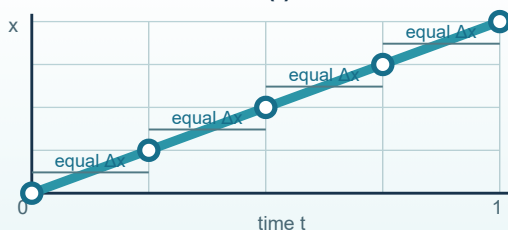
$N = 1, 2, 4, \dots$ divides that same second more finely.

Each duration changes

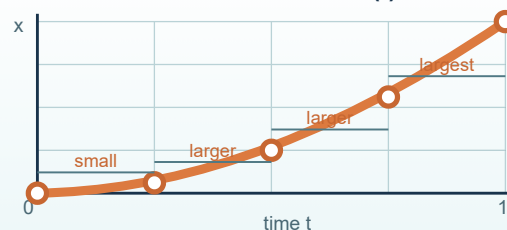
For equal slices, $\Delta t = 1/N$.

If a proposed action changes only because the camera took more frames, it is measuring the recording scheme—not only the motion.

STEADY HISTORY · $x(t) = t$



ACCELERATING HISTORY · $x(t) = t^2$



Equal time slices reveal how the speed changes along a history.

PAGE 1 · TEST THE TEMPTING SCORE

Complete the tables here. Show calculus work on separate paper.

- First test the tempting score $Q_N = \sum (\Delta x_k)^2$ on the steady history $x(t) = t$. Complete the table. The physical motion is unchanged, so explain why the tempting score cannot be its action.

N	Δt	Each Δx_k	Q_N
1	1	1	_____
2	1/2	1/2	_____
4	1/4	1/4	_____



ACTION BETWEEN FRAMES

$$S_N = \sum_{k=1}^N \left(\frac{\Delta x_k}{\Delta t} \right)^2 \Delta t$$

Each interval contributes **speed squared × duration**. The factor Δt accounts for how long that speed lasts.

- Now use S_N for $x(t) = t$. Show in one line that the corrected score equals 1 for every N . Then complete the accelerating-history table for $x(t) = t^2$.

N	Δt	Interval speeds $\Delta x_k / \Delta t$	S_N
1	1	1	_____
2	1/2	1/2, 3/2	_____
4	1/4	1/4, 3/4, 5/4, 7/4	_____

- In the limit of infinitely fine time slices, the sums become

$$S[x] = \int_0^1 \left(x'(t) \right)^2 dt.$$

Compute both histories' continuous actions. Compare them with the frame sums. Explain what changes under finer measurement and what stays attached to the history.

Normalization note: A fixed positive factor does not change comparisons; $\Delta t = 1$ hid it in Lab 1.

