



The Safe Zone



COMPLETE TO COLLECT

Emerald Crystal

Level 2 · 4 Glints

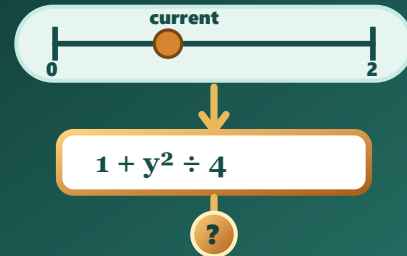
CAN ONE CHECK CONTROL EVERY STEP?

Start at $y_0 = 0$ and repeat the recurrence

$$y_{n+1} = 1 + \frac{y_n^2}{4}.$$

An interval is **invariant under a rule** if every current value in that interval produces a next value in the same interval. In plain language: **once the process is in the safe zone, the rule cannot send it out.**

CANDIDATE SAFE ZONE



Test the whole zone, not just a few sample values.

WHAT AN INVARIANT INTERVAL PROOF MUST DO

Assume the current value is anywhere in I



apply the rule to the whole range



prove the next value is still in I

Examples are clues

A few calculated values can suggest a safe zone, but they do not cover every possible current value.

Bounds are a certificate

An inequality that starts with an arbitrary value in the interval proves one safe step for all of them.

The repeat engine: start inside + prove every step stays inside = every later value stays inside.



- Calculate y_1, y_2, y_3 exactly. Do these examples suggest that $[0, 2]$ may be a safe zone?

- Now test **every possible current value** in the interval. Assume $0 \leq y_n \leq 2$ and complete the chain. Explain why the final bounds keep the next value inside $[0, 2]$.

$$0 \leq y_n^2 \leq \underline{\hspace{2cm}} \quad 0 \leq \frac{y_n^2}{4} \leq \underline{\hspace{2cm}} \quad \underline{\hspace{2cm}} \leq y_{n+1} \leq \underline{\hspace{2cm}}$$

- The process starts at $y_0 = 0$. Use that fact and Part 2 to explain why **every** later value stays in $[0, 2]$, even though you cannot list infinitely many values.

- Is the smaller interval $[0, \frac{3}{2}]$ invariant? Test the allowed current value $y_n = \frac{3}{2}$ and justify your answer.

