



The Orbit of i



COMPLETE TO COLLECT
Smoky Quartz Point
Level 2 · 4 Glints



PAGE 1 · BUILD THE TOOLS COMPLEX NUMBERS, ONE IDEA AT A TIME

The new number i is defined by $i^2 = -1$. A **complex number** has the form $a + bi$, with a real part a and an i -part b .

THREE MOVES YOU WILL USE

1 Turn a number into a point

Plot $a + bi$ at (a, b) . For example, $-2 + 3i$ goes at $(-2, 3)$.

2 Square by expanding

$$\begin{aligned}(a + bi)^2 &= a^2 + 2abi + b^2i^2 \\ &= (a^2 - b^2) + 2abi\end{aligned}$$

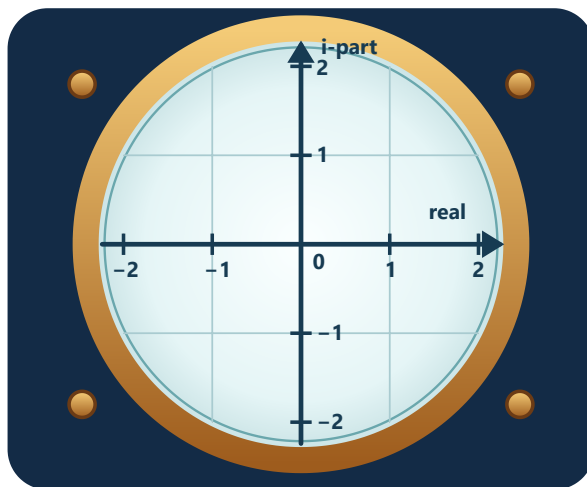
The last step uses $i^2 = -1$. Example:
 $(1 + i)^2 = 1 + 2i + i^2 = 2i$.

3 Measure distance from 0

The point (a, b) is $\frac{|a + bi|}{\sqrt{a^2 + b^2}}$ units from 0, by the Pythagorean theorem.



Start with $z_0 = 0$ and repeat $z_{n+1} = z_n^2 + c$. This list of points is an **orbit**. If an orbit repeats while every point has distance at most 2 from 0, it stays bounded and c belongs to the Mandelbrot set.



Plot each distinct orbit point in the lens.

- For $c = i$, calculate z_1, z_2, z_3, z_4 . Show enough squaring to make each step checkable, complete the table, and plot each distinct point in the lens.

n	z_n	Point (a, b)
0	0	(0, 0)
1		
2		
3		
4		

- Which values form a repeating cycle? Use the recurrence to predict z_5 and z_6 .

- Find the distance from 0 of each value in the cycle. Explain why this proves that $c = i$ belongs to the Mandelbrot set.

