



Rook's Halfway Signal



COMPLETE TO COLLECT

Emerald Crystal

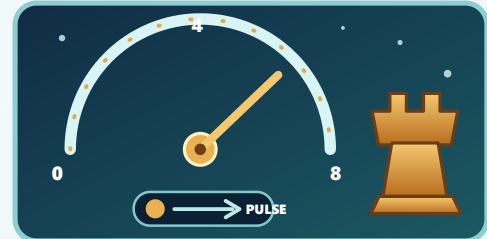
Level 2 · 4 Glints

ROOK'S SIGNAL

Rook is tuning an observatory dial toward a target reading of **8**. The dial starts at $x_0 = 0$. After each pulse, its new reading is halfway between its current reading and 8:

$$x_{n+1} = \frac{x_n + 8}{2}$$

This is a **recurrence**: a rule that uses the current value to make the next value. The subscript tells the step number, so x_3 is the reading after 3 pulses.



Each pulse makes one new reading.

READ A RECURRENCE ONE STEP AT A TIME

A subscript is a step label, not multiplication. In x_n , the n tells which reading you have. The label $n + 1$ means the next reading.

Practice rule: $q_0 = 10, \quad q_{n+1} = q_n - 2$

Current: $q_0 = 10$

Substitute: $q_1 = q_0 - 2$

Next: $q_1 = 8$

Current: $q_1 = 8$

Substitute: $q_2 = q_1 - 2$

Next: $q_2 = 6$

Rook's rule works the same way: use one known reading to make exactly one next reading.



- Use the recurrence to complete the table.

n	Reading x_n	Gap to the target $8 - x_n$
0	0	
1		
2		
3		
4		
5		

- Describe what happens to the gap after each pulse. Then write a rule for the gap after n pulses.

- Find x_{10} **without** calculating x_6, x_7, x_8, x_9 one at a time. Explain how the gap pattern helps.

