



One Step Past Omega

**COMPLETE TO COLLECT****Tumbled Jade**

Level 2 · 4 Glints

PAGE 1 · BUILD THE IDEA ROWS HAVE ORDER TYPES

Ordinals describe positions in order. To form $\alpha + \beta$, place the entire row for α first, then the entire row for β . The first infinite ordinal is ω : 0, 1, 2, 3, It has a first position and a next position after every position, but no last position. Two rows represent the same ordinal when their positions can be paired in order.

REHEARSE WITH FINITE ROWS

Same order type

• • • and ★ ★ ★ both have three positions.
Pair first with first, second with second, and third with third.

Add by joining

$$2 + 3 = \underbrace{\bullet \bullet}_2 \underbrace{\bullet \bullet \bullet}_3 = 5$$

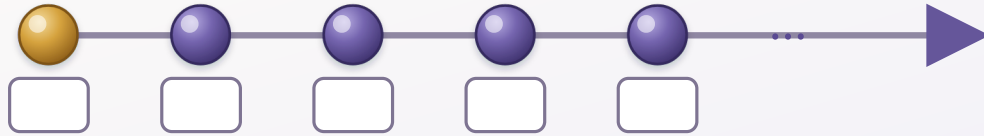
For finite rows, reversing the two lengths still gives five positions.

Now remove the last position

The missing last position in ω is why the order of the addends may matter.



$1 + \omega$



$\omega + 1$



Write the first five matching labels in the boxes beneath the top strand.

**PAGE 2 · FOLLOW
THE INFINITE ORDER**

Write on the diagram. Use scratch paper
for explanations.

- On the $1 + \omega$ strand, write 0 under the first bead, then 1, 2, 3, and 4 under the next four. Describe how the labeling continues. Does this pair every position with ω in order? Find $1 + \omega$.
- On the $\omega + 1$ strand, the gold bead comes after every position in the copy of ω . Does this strand have a last position? Does ω ? Use that difference to decide whether $\omega + 1 = \omega$.
- An operation is **commutative** if swapping the addends never changes the result. Is ordinal addition commutative? Write an equation or inequality from Parts 1 and 2 as evidence.
- Without drawing another infinite strand, decide whether $3 + \omega = \omega$ and whether $\omega + 3 = \omega$. Explain both decisions.

