



Knightmare



COMPLETE TO COLLECT

The Knightmare
Paraíba

Unique Jewel · 8 Glints

THE PROBLEM

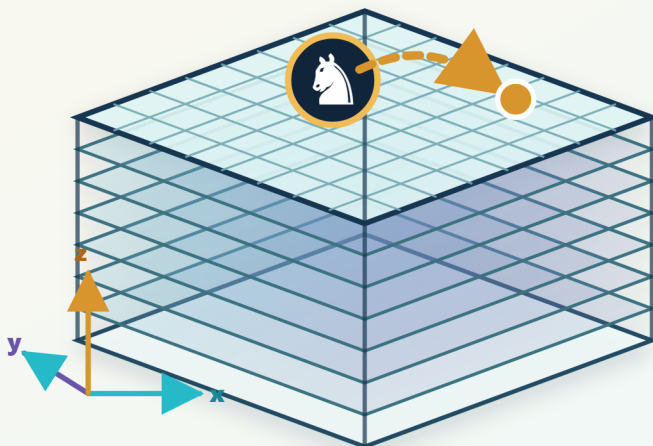
A **chess knight** moves through a glass cube divided into $8 \times 8 \times 8 = 512$ **rooms**. Each room has an address **(x, y, z)**, where x, y, and z are each whole numbers from 1 through 8.

In one move, the knight changes **exactly two different coordinates**: one changes by exactly **2**, the other changes by exactly **1**, and the third does not change. Each change may add or subtract. A destination counts only if all three of its coordinates remain from 1 through 8. Two moves count as different only when they land in different rooms.

A room's **move number** is the number of different rooms the knight can reach from it in one move.

THE GLASS CUBE

8 layers $\times$ 8 rows $\times$ 8 columns = 512 rooms



ADDRESS LOCK

Every room is (x, y, z)

x: 1–8

y: 1–8

z: 1–8

ONE MOVE

Change the coordinates by

2

1

0

in any
order

The 2 and 1 may each be
added or subtracted.



(4, 4, 4) $\rightarrow$ (6, 3, 4)



(4, 4, 4) $\rightarrow$ (6, 3, 5)
changes all three coordinates



THE INVESTIGATION

Organize your cases and proof on separate paper.

- Determine **every move number** that occurs somewhere in the cube. Give at least one example room address for each move number you claim is possible.
- Determine how many of the 512 rooms have each possible move number. Which move number is most common?
- Prove that your classification is complete **without checking 512 rooms one at a time**. Your proof must explain why every room is covered, why no destination is counted twice, and why your room totals add to 512.

YOUR PROOF MUST UNLOCK ALL THREE SEALS

◆ Every room is covered.

◆ No destination is counted twice.

◆ Your room totals add to 512.

