



Everything Points Here



COMPLETE TO COLLECT
Sunstone Cabochon
Level 3 · 6 Glints

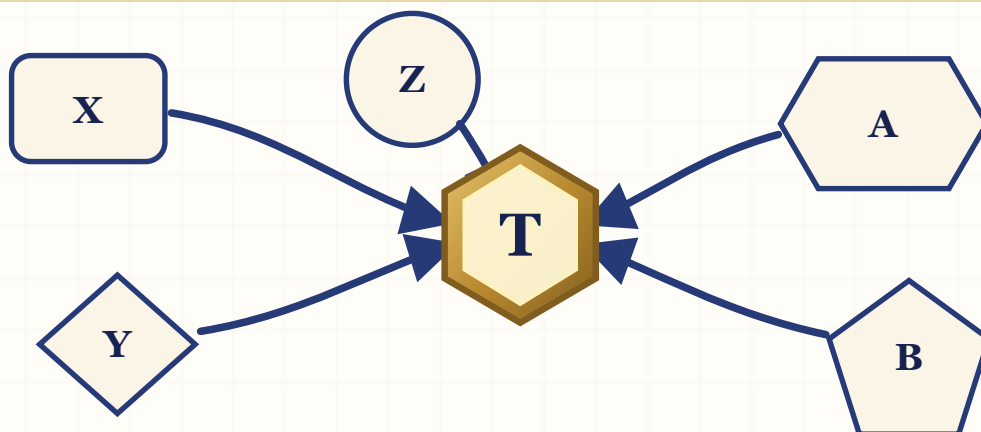
MEDITATIONS ON ONENESS **ARCHIVE FOLIO** |||||

ONE DESTINATION

An Archive folio records a formal system of objects and arrows. It begins with sets and functions, then asks what any terminal object must force.

*“Do not ask first
whether they are
equal. Ask what
arrows must exist.”*

— Vivian



A FORMAL SYSTEM OF ARROWS

A **category** consists of objects and arrows. An arrow $f : X \rightarrow Y$ goes from object X to object Y .

If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$, their composite is $g \circ f : X \rightarrow Z$. Every object X has an identity arrow $\text{id}_X : X \rightarrow X$. Identity arrows change nothing, and composition is associative.

The category **Set** has sets as objects and functions as arrows.

An object T is **terminal** if, for every object X , there is exactly one arrow $X \rightarrow T$.

1. TEST THE DEFINITION IN SETS

Use separate paper for your solution.

Let $A = \{r, s, t\}$, $1 = \{\star\}$, **and** $B = \{0, 1\}$.

- How many functions are there from A to each of $\emptyset$, 1 , and B ? Explain each count.**
- Prove that 1 is terminal in **Set**. Include the case $X = \emptyset$.**
- Use $X = \{r\}$ to prove that neither $\emptyset$ nor B is terminal.**

There is exactly one function from the empty set to any set: it has no inputs to assign.



WHEN TWO OBJECTS ARE THE SAME IN STRUCTURE

An arrow $f : T \rightarrow U$ is an **isomorphism** if there is an arrow $g : U \rightarrow T$ such that

$$g \circ f = \text{id}_T, \quad f \circ g = \text{id}_U.$$

Then f and g are inverses, and T and U are isomorphic.

YOUR ARROW DIAGRAM

Draw and label the forced arrows. Add the two composite loops you use in your proof.



2. PROVE THE THEOREM

Mark the diagram here. Use separate paper for the proof.

Suppose T and U are both terminal objects in the same category.

- Draw and label the unique arrow $f : T \rightarrow U$ and the unique arrow $g : U \rightarrow T$.
- Prove that f and g are inverses.
- Prove that f is the only isomorphism from T to U .

Your conclusion should establish the full statement:

