



# The Same Equation, Different Worlds



**COMPLETE TO COLLECT**  
**Tanzanite Crystal**  
 Level 2 · 5 Glints

## MEDITATIONS ON ONENESS · LOCAL FOLIO MARK 11111111

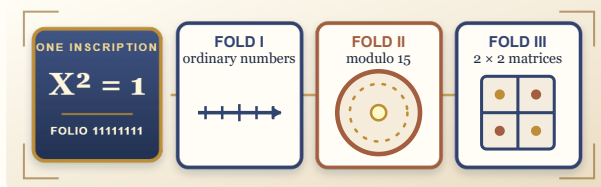
### ONE INSCRIPTION, THREE FOLDS

Meditation 8 preserves one proof copied into three internally coherent folds of the Manyfold. A margin claim says that  $X^2 = 1$  always has only the two solutions  $X = 1$  and  $X = -1$ .

*"The inscription survived. Which inference did not?"*  
 — Vivian Meridian



Vivian compares the panels before accepting the claim.



Same inscription. Three mathematical structures.

## THE DISPUTED PROOF

$$X^2 = 1 \quad \rightarrow \quad (X - 1)(X + 1) = 0 \quad \rightarrow \quad X - 1 = 0 \quad \text{or} \quad X + 1 = 0$$

The final arrow uses the **zero-product rule**:

$$PQ = 0 \quad \Rightarrow \quad P = 0 \text{ or } Q = 0.$$

## FOLD I · ORDINARY NUMBERS

Use separate paper for the proof and check.

- The zero-product rule is true for integers and for complex numbers. Use the Archive argument to find all integer solutions and all complex-number solutions of  $X^2 = 1$ . Check that every solution you list works, and explain why there can be no others.



## FOLD II · ARITHMETIC ON FIFTEEN POSITIONS

### THE 15-POSITION DIAL

On this dial, calculations are made modulo 15. Here  $-1$  is the same position as 14.

Meditation 5 records

$$x^2 \equiv 1 \pmod{15}$$

at exactly  $x = 1, 4, 11, 14$ .

### AUDIT THE PROOF AT $x = 4$

Record the audit on this page.

- Use  $x = 4$  to audit the Archive proof.

| Check                      | Your calculation or conclusion |
|----------------------------|--------------------------------|
| Remainder of $4^2$         |                                |
| $4 - 1$ modulo 15          |                                |
| $4 + 1$ modulo 15          |                                |
| Product of the two factors |                                |
| Which arrow fails?         |                                |

Show that the factor product is 0 modulo 15 although neither factor is 0. Identify the precise arrow in the Archive proof that fails.



### FOLD III · MATRIX ARITHMETIC

#### THE MATRIX WORLD

For  $2 \times 2$  matrices, use

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} ap + br & aq + bs \\ cp + dr & cq + ds \end{pmatrix}.$$

The matrix versions of 1 and 0 are

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

For every real number  $t$ , define

$$A_t = \begin{pmatrix} 1 & t \\ 0 & -1 \end{pmatrix}.$$

#### AN INFINITE FAMILY

Use separate paper for the matrix products and proof.

- **Prove all of the following.**
  - $A_t^2 = I$  for every real  $t$ .
  - $A_t \neq I$  and  $A_t \neq -I$ .
  - Different values of  $t$  give different matrices, so  $X^2 = I$  has infinitely many matrix solutions.
  - Both  $A_t - I$  and  $A_t + I$  are nonzero, but  $(A_t - I)(A_t + I) = O$ .

**Finish with one sentence: name the step that stayed valid in all three folds, and the inference that did not.**

