



Can There Be Two Ones?



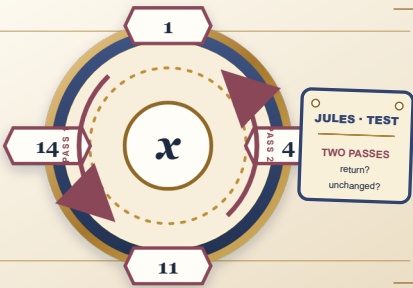
COMPLETE TO COLLECT
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Level 3 · 5 Glints

MEDITATIONS ON ONENESS · LOCAL FOLIO MARK 111111
JULES'S FOUR TEST GATES

On a 15-position Observatory dial, the multiplier machine M_s sends position x to the remainder of sx after division by 15.

The four labels $s = 1, 4, 11, 14$ all satisfy

$$s^2 \equiv 1 \pmod{15}.$$



Jules can test a return in two passes. Identity makes a stronger promise.

TWO DIFFERENT PROMISES

Self-undoing

$$M_s(M_s(x)) \equiv s^2x \equiv x \pmod{15}$$

Two passes return every input.

Identity

$$M_s(x) = x \quad \text{for every } x$$

One pass leaves every input unchanged.

SEPARATE THE PROMISES

Complete the table here. Use scratch paper as needed.

- Complete the table. Which machine is the identity machine? Prove that your choice works for every x , and use the input $x = 1$ to rule out the other three.

s	$M_s(1)$	Identity machine?
1		
4		
11		
14		



JULES'S RETURN TEST

Use scratch paper as needed.

- **Choose one non-identity machine. Write a two-pass journey $x \rightarrow M_s(x) \rightarrow x$ in which the first pass changes x . Explain why “self-undoing” and “identity” do not mean the same thing.**

VIVIAN'S ARCHIVE NOTE · FORGET THE DIAL
ANY OPERATION. ANY OBJECTS.

Let $\star$ be any operation that combines two objects in a set. An object I is an **identity** when

$$I \star X = X \star I = X \quad \text{for every object } X.$$

“If two labels make the same claim, compare the claim—not the labels.” — Vivian Meridian

THE UNIQUENESS PROOF

Use separate paper for your proof.

Suppose A and B both satisfy the identity rule. Prove that $A = B$ using only that rule.
Do not assume that the objects are numbers or that $\star$ is multiplication.

