



Four Ways Back to One



COMPLETE TO COLLECT

Green Tourmaline

Level 2 • 4 Glints

MEDITATIONS ON ONENESS • LOCAL FOLIO MARK 11111 ROOK'S SPLIT DIAL

At the Observatory, Rook tests a dial whose positions are numbered 0 through 14. To square a position, multiply it by itself and keep only the remainder after division by 15.

We write $a \equiv b \pmod{n}$ when a and b have the same remainder after division by n .



THE RETURN CONDITION

Complete the folio: find every dial position x for which

$$x^2 \equiv 1 \pmod{15}.$$

Since $15 = 3 \cdot 5$, every solution must also satisfy

$$x^2 \equiv 1 \pmod{3} \quad \text{and} \quad x^2 \equiv 1 \pmod{5}.$$

MAKE THE TWO LOCAL RECORDS

Complete and mark both tables on this page.

- Complete both square-remainder tables. On each dial, circle every input whose square lands at 1.

x on the 3-dial	0	1	2
Remainder of x^2			

x on the 5-dial	0	1	2	3	4
Remainder of x^2					



PAGE 2 · RECOMBINE THE CLUES
FOUR POSSIBLE PAIRS

Rook's run record gives each position one remainder on the 3-dial and one on the 5-dial. Every return must choose one circled remainder from each smaller dial.

COMPLETE THE RETURN MAP

Use the table here. Use separate paper for the proof.

- Complete the four rows. In each row, find every x from 0 through 14 having both stated remainders. Then list all solutions of $x^2 \equiv 1 \pmod{15}$.

Remainder of $x \bmod 3$	Remainder of $x \bmod 5$	x from 0 through 14
1	1	
1	4	
2	1	
2	4	

- Check every solution by writing $x^2 = 15Q + 1$ for a whole number Q . Then explain why the four-row table proves that there are no other solutions.

