



The Empty Arrangement



COMPLETE TO COLLECT

Indicolite

Level 3 · 5 Glints

MEDITATIONS ON ONENESS

LOCAL FOLIO MARK · 1111

PAGE 1 · INSPECT THE CASE WITH NOTHING IN IT

THREE GALLERY PLATES

The Gallery is preparing one plate for two seals, one for a single seal, and one for none. A **complete arrangement** of n distinct seals places all n seals into n numbered cases, one seal per case, using every seal.

Let $n!$ be the number of complete arrangements.

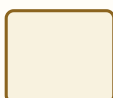
two seals · two cases



case 1



case 2



arrangement 1: _____

arrangement 2: _____

one and one



case 1



arrangement: _____

zero and zero

no seals
no cases



complete arrangement? _____

Use the first two panels to list arrangements. Decide whether the untouched third panel is complete.



THE UNTOUCHED DISPLAY

Write arrangement lists on the diagram. Use scratch paper for the proof.

- List every complete arrangement of two seals A, B in two numbered cases. Count the complete arrangements of one seal A in one case. Now consider zero seals and zero cases. Does the untouched display count as a complete arrangement? Prove that there is exactly one complete arrangement—not zero arrangements and not more than one.

$$2! = \underline{\hspace{2cm}}$$

$$1! = \underline{\hspace{2cm}}$$

$$0! = \underline{\hspace{2cm}}$$

PAGE 2 • FOLLOW THE EMPTY ECHO TWO INDEPENDENT CHECKS

Choose the first case

For $n \geq 1$, choose one of the n seals for case 1, then arrange the remaining $n - 1$ seals.

Record arrows

A **permutation** of a collection S sends every object to one destination in S , with no repeated or missing destinations.

CHECK THE COUNT TWICE

Use separate paper for both proofs.

- Use the first-case choice to justify $n! = n(n - 1)!$ for $n \geq 1$. Substitute $n = 1$ to determine $0!$. Explain why this agrees with Part 1. Compare your result with this rule: a product containing no factors has value 1.
- The **identity permutation** sends every object to itself. Prove that every collection has exactly one identity permutation. Describe its instruction list when S is empty.

Nothing is placed. Nothing is omitted. One arrangement is complete.

