



The Infinite Factorization Glitch

**COMPLETE TO COLLECT****Tumbled Hematite**

Level 2 · 4 Glints

MEDITATIONS ON ONENESS

LOCAL FOLIO MARK · 11

PAGE 1 · FIND THE MISSING STOP**ROOK'S UNFINISHED RULE CARD**

Rook is testing an old Archive instruction. It begins with this draft definition:

A positive whole number is a **prime candidate** if it cannot be written as a product of two smaller positive whole numbers.

Under this draft definition, 1 is a prime candidate: there are no smaller positive whole numbers available to multiply.

Whenever a factor label for 30 works, Rook may stamp a longer label if he can add another prime candidate without changing the product. The instruction contains no stopping rule.

ARCHIVE**exactly 3 factors**

$$30 = \boxed{} \times \boxed{} \times \boxed{}$$

exactly 11 factors

$$30 = \boxed{} \times \boxed{} \times \boxed{} \times \boxed{} \times \boxed{} \times \boxed{} \times \boxed{} \times \boxed{} \times \boxed{} \times \boxed{} \times \boxed{}$$

factor labels

How long can the label become?

Write one factor in every compartment. Every factor must pass the draft definition.



STRETCH THE LABEL

Write factors in the diagram. Use scratch paper for your explanation.

- Under the draft definition, write 30 as a product of exactly 3 prime candidates and as a product of exactly 11 prime candidates. Then explain how to write 30 as a product of exactly k prime candidates for every whole number $k \geq 3$, and explain why this proves that Rook never reaches a last valid label.

PAGE 2 • REPAIR THE RULE

SEPARATE UNITS FROM PRIMES

ROOK'S RUN RECORD STOP CONDITION MISSING

Now widen the number system: every definition below is about the integers, so negative factors are allowed.

UNIT

An integer u is a **unit** if some integer v satisfies $uv = 1$.

ASSOCIATES

Integers a and b are **associates** if $a = ub$ for some unit u .

IRREDUCIBLE

A nonzero nonunit integer q is **irreducible** if $q = ab$ forces a or b to be a unit.

Granted for this problem: $\pm 2, \pm 3, \pm 5$ are irreducible.

REPAIR THE COMPARISON

Use separate paper for your proofs and explanation.

- Work now in the integers. Prove that the only units are 1 and -1 . Then let p be any positive prime. Prove that p and $-p$ are associates.
- Compare the two irreducible factorizations $30 = 2 \cdot 3 \cdot 5$ and $30 = (-2)(-3) \cdot 5$. Explain why changing order alone is not enough and how associates repair the comparison. Then read the Integer Factorization Theorem below. Explain why your examples motivate its qualifiers but do not prove either existence or uniqueness.

Integer Factorization Theorem

Every nonzero integer that is not a unit is a finite product of irreducible integers. If two such products have the same value, then they have the same number of factors and, after reordering, corresponding factors are associates.

Integer units:

Uniqueness is up to order and

