



The Machine That Repeats Forever



COMPLETE TO COLLECT

Tumbled Sunstone

Level 2 · 4 Glints

MEDITATIONS ON ONENESS

LOCAL FOLIO MARK · 1

PAGE 1 · LET THE MACHINE PASS THROUGH ITSELF**ROOK'S ENDLESS TEST**

In the Observatory, Rook feeds each output back through the same multiplication machine. The machine accepts any nonnegative whole number, and every pass multiplies its input by the same hidden nonnegative whole number c . Write $M(x) = cx$.



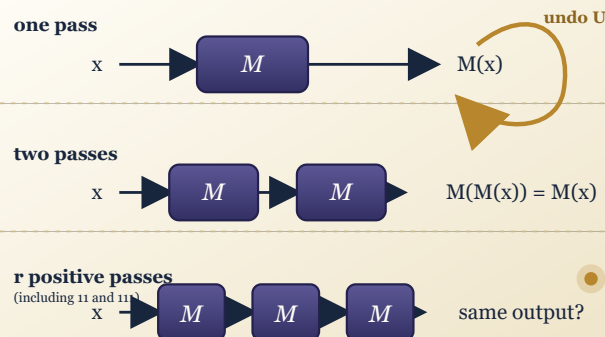
Rook can make the chain as long as requested. Your proof must account for every pass.

Reversible

An undo machine U recovers every input, so $U(M(x)) = x$ for every x .

Repeat-stable

Two passes have exactly the same effect as one: $M(M(x)) = M(x)$ for every x .



One pass and two passes agree. What does that force every longer chain to do?

FOLLOW THE REPETITIONS

Use separate paper for your proof.



- Without finding c , prove that r passes have exactly the same output as one pass for every positive whole number r . State what this says about 11 passes and 111 passes.

PAGE 2 · FIND THE HIDDEN MULTIPLIER

TWO CANDIDATES, ONE REVERSIBLE SURVIVOR

Now use that c is a nonnegative whole number. For the input 1, repeat-stability says that the one-pass and two-pass outputs agree.

Trace the input 1 through both routes.

$$M(M(1)) = M(1)$$

LET REVERSIBILITY DECIDE

Use separate paper for your proof.

- Use the input 1 to prove that $c \cdot c = c$. Prove that c is 0 or 1 by explaining why $c \cdot c > c$ when $c \geq 2$. Then rule out $c = 0$: show that the different inputs 1 and 2 would both produce 0, so one undo machine could not recover both inputs. Conclude that $c = 1$.

The hidden multiplier is _____

After the first pass, every later pass is only an echo.

